Grade 11/12 Math Circles
November 8, 2023
P-adic numbers, Part 2 - Problem Set

1. Find all solutions to $x^2 = 1 \bmod 11$ and $\bmod 13$.

2. For $p = 3$, $x = 36$, we have that $x = 1100_3$. Find $|x^2|_p$.

3. Show that if x is a rational number then the product of all the numbers $|x|_p$ for p a prime is 1.

4. Show that $d(x, z) = |x - z|_p \leq d(x, y) + d(y, z)$ for p-adic numbers. Better still, show that $d(x, z) \leq \max d(x, y), d(y, z)$.